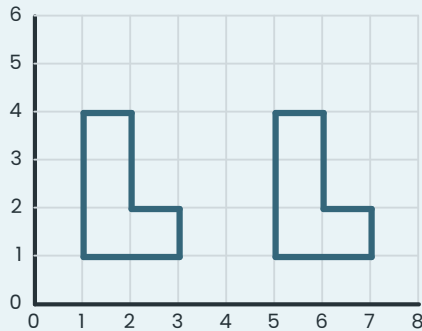


# Spatial Reasoning Missions: Turn, Flip, Fold, and Navigate

6 complete activities from Spatial Reasoning Missions, each with a worked answer.



## HOW TO USE

1. Read the situation and the evidence first.
2. Show your thinking in the work boxes, not only an answer.
3. Explain how you know your answer makes sense.
4. Try the new problem at the bottom of the page.

6 activities • 3 answer pages • From the 24-activity collection

10 US Letter pages • Print at 100% / Actual Size

Original Bobotemp educational printable • Fictional names, places and prices

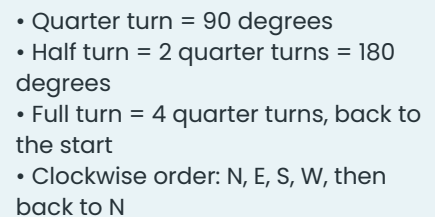
Name \_\_\_\_\_ Date \_\_\_\_\_

## LEVEL 1



Use the compass rose: N is up, E is right, S is down, and W is left. Clockwise means the way clock hands move. Hold a pencil as the arrow and turn it as you read each step of the code.

A signal arrow starts pointing North. Its turn code is: a quarter turn clockwise, then a half turn clockwise, then another half turn clockwise. Track where the arrow points after each turn. How many quarter turns clockwise did the arrow make in all? What one short turn would do the same job as the whole code?



1. Make a table showing where the arrow points after each turn.

2. Write an addition sentence for the total number of quarter turns.

[illegible]

### 3. Explain why one short turn can do the same job as the whole code.

[illegible]

TRY IT • An arrow starts pointing South. It makes 6 quarter turns clockwise. Where does it point? What smaller turn does the same job?

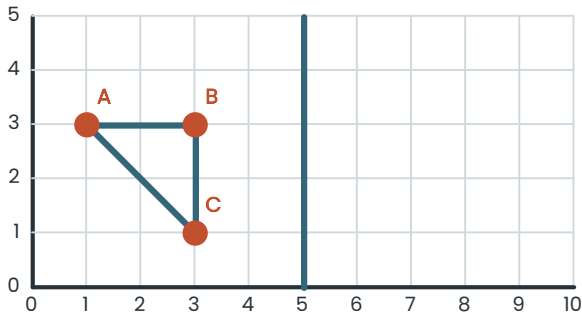
Name \_\_\_\_\_ Date \_\_\_\_\_

## LEVEL 1



The mirror line is the vertical line  $x = 5$ . Count how far each point is from the line. Then count the same distance on the other side. Points move straight across, never up or down.

A 3-corner shape has corners  $A(1, 3)$ ,  $B(3, 3)$ , and  $C(3, 1)$ . Reflect the shape across the vertical mirror line  $x = 5$ . Find the image of each corner. Then check: what is the x-coordinate of the image of  $A$ ?



## EVIDENCE TO USE

- Mirror line:  $x = 5$  (vertical)
- Corners: A(1, 3), B(3, 3), C(3, 1)
- Same distance from the line on both sides
- The y-coordinate stays the same

1. Count the units from each corner to the mirror line.

[illegible]

2. Plot each image point on the other side of the line.

A blank sheet of graph paper with a grid pattern. The grid consists of 10 columns and 8 rows of squares. The paper has rounded corners and a light gray border.

3. Explain why the y-coordinates did not change.

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**TRY IT •** Reflect the point  $(8, 2)$  across the same line,  $x = 5$ .

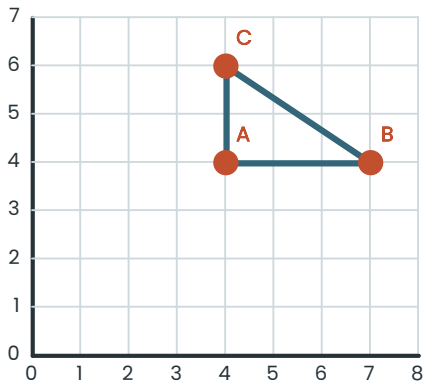
Name \_\_\_\_\_ Date \_\_\_\_\_

LEVEL 2 ●●○

The center of rotation is A(4, 4), and it stays still. Describe each other vertex as a distance and direction from the center. Then turn that direction clockwise and keep the same distance.

Triangle ABC has vertices A(4, 4), B(7, 4), and C(4, 6). Rotate the triangle a quarter turn (90 degrees) clockwise about the marked center point A(4, 4). Record the coordinates of A', B', and C'. Check: what is the y-coordinate of B'?

QUARTER TURN CLOCKWISE ABOUT THE CENTER A



## EVIDENCE TO USE

- Center: A(4, 4)
- Turn: 90 degrees clockwise
- Clockwise: east becomes south
- Clockwise: north becomes east
- Distances from the center stay the same

1. Describe B and C as a distance and direction from A.

[illegible]

2. Turn each direction clockwise and plot the image points.

[illegible]

3. Explain how you know the image is the same size and shape.

[illegible]

TRY IT • Rotate the same triangle 90 degrees counterclockwise about A(4, 4). Where do B and C land?

Name \_\_\_\_\_ Date \_\_\_\_\_

## LEVEL 2



The grid is 5 by 5. Name each square by (column, row). Columns 1 to 5 go left to right, and rows 1 to 5 go bottom to top. Partner A reads the directions aloud. Partner B draws without peeking.

Jordan described a hidden 4-square design: Put a square in the middle. Put one next to it. Put one above that. Put one in a corner. Four designs fit some of these words. W: (3,3), (4,3), (4,4), (5,5). X: (3,3), (2,3), (2,4), (1,5). Y: (3,3), (4,3), (4,4), (1,1). Z: (3,3), (3,2), (3,1), (5,1). Jordan then wrote exact clues. Clue 1: The third square is directly above the second square. Clue 2: The second square is at (4, 3), 1 unit right of (3, 3). Clue 3: One square is at the corner (1, 1). Which design is hidden?

## DESIGN X

## DESIGN Z



- Squares are named (column, row)
- Row 1 is the bottom row
- Vague words: next to, in a corner
- Exact words: a direction plus a distance, or a coordinate

1. Circle each vague word in Jordan's first description.

2. Test each design against the exact clues, in order.

[illegible]

3. Rewrite one vague sentence so it gives exactly one answer.

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**TRY IT •** Draw this design: Start at (1, 2). Put the next square 1 unit right of it. Put the next square 2 units up from that one. Put the last square in the top-right corner. List all four squares.

Name \_\_\_\_\_ Date \_\_\_\_\_

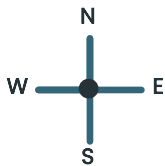
### LEVEL 3



Add up all the clockwise turns. Then subtract the counterclockwise turns, because turning back undoes clockwise turning. Confirm by tracking the arrow one move at a time.

A signal arrow starts pointing west. It makes three moves: 270 degrees clockwise, then 180 degrees counterclockwise, then 90 degrees clockwise. What single clockwise turn does the same job? Where does the arrow end up pointing?

## THE THREE MOVES



- 1 Start: pointing west
- 2 270 degrees clockwise
- 3 180 degrees counterclockwise
- 4 90 degrees clockwise

- Clockwise: N, E, S, W
- Counterclockwise: N, W, S, E
- 270 degrees clockwise = 90 degrees counterclockwise
- 180 degrees = a half turn

1. Write one number sentence that adds the clockwise turns and subtracts the counterclockwise turn.

[illegible]

2. Track the arrow after each move to confirm.

[illegible]

3. Explain why 270 degrees clockwise and 90 degrees counterclockwise match.

[illegible]

**TRY IT •** Show that 270 degrees clockwise and 90 degrees counterclockwise send an arrow pointing east to the same direction.

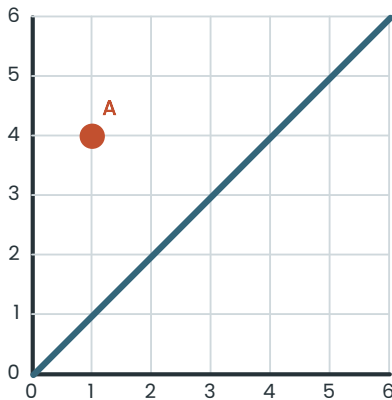
Name \_\_\_\_\_ Date \_\_\_\_\_

### LEVEL 3



The mirror is the diagonal line through  $(0, 0)$ ,  $(1, 1)$ ,  $(2, 2)$ , and so on. To reflect a point, travel along a path that crosses the mirror at a right angle. Each step goes 1 right and 1 down, or 1 left and 1 up. Count the total steps it takes to cross the mirror and end the same distance past it. Then check that the halfway point of your path lies on the mirror. The halfway point may fall between grid points.

Point A(1, 4) is reflected across the diagonal mirror. Four students give answers: (4, 1), (1, 4), (3, 2), and (6, 3). Clue 1: A is above the diagonal, so its image is below it, where  $x$  is greater than  $y$ . Clue 2: Each step changes  $x$  and  $y$  by equal and opposite amounts, so  $x + y$  stays 5. Clue 3: A is 3 more up than across ( $4 - 1 = 3$ ), so its image must be 3 more across than up ( $x - y = 3$ ). Which answer is correct?



- Mirror: the diagonal line through  $(0, 0)$  and  $(6, 6)$
- Point:  $A(1, 4)$
- Step: 1 right and 1 down
- Equal distance on both sides of the mirror

### 1. Test each student answer against every clue.

[illegible]

2. Trace the step path from A to its image and mark the halfway point on the mirror.

[illegible]

3. Explain why a sideways path does not work for this mirror.

**TRY IT • Reflect B(2, 6) across the same diagonal mirror.**

## 1. Arrow Turn Code

LEVEL 1

The arrow ends pointing East. It made 5 quarter turns clockwise, because  $1 + 2 + 2 = 5$ . A single quarter turn clockwise does the same job, because 4 of the 5 quarter turns make a full turn.

**WORKED REASONING** • Track the arrow. Start: North. The quarter turn clockwise sends North to East. The half turn points the opposite way, so East becomes West. The second half turn sends West back to East. Count the quarter turns. The quarter turn counts as 1, and each half turn counts as 2, so  $1 + 2 + 2 = 5$ . Four quarter turns make a full turn, which brings the arrow back to where it started. That leaves  $5 - 4 = 1$  quarter turn, and one quarter turn clockwise sends North to East. Both methods give East.

**WATCH FOR** • Some students count each half turn as only 1 quarter turn. They get 3 quarter turns, which would leave the arrow pointing West. A half turn is 2 quarter turns. Other students expect so much turning to send the arrow somewhere new, but every 4 quarter turns bring it back to the start.

**TRANSFER** • North. Six quarter turns are 4 quarter turns (a full turn) plus 2 more (a half turn). So the code does the same job as a half turn, and South turns to North.

## 2. Mirror Match on a Dot Grid

LEVEL 1

A' is (9, 3), so the checked x-coordinate is 9. B' is (7, 3) and C' is (7, 1).

**WORKED REASONING** • A is at  $x = 1$ , which is  $5 - 1 = 4$  units left of the line. Count 4 units right of the line:  $5 + 4 = 9$ , so A' is (9, 3). B and C are at  $x = 3$ , which is 2 units left of the line. Their images are 2 units right of the line, at  $x = 7$ : B' is (7, 3) and C' is (7, 1). A vertical mirror moves points only left or right, so every y-coordinate stays the same.

**WATCH FOR** • Some students slide the shape 5 units right instead of flipping it, which puts A at (6, 3). A slide keeps the shape facing the same way. A reflection keeps each point the same distance from the mirror line, so the point farthest left lands farthest right.

**TRANSFER** • (2, 2). The point is 3 units right of the line, so its image is 3 units left of the line:  $5 - 3 = 2$ .

## 9. Quarter Turns on a Grid

LEVEL 2

$A'(4, 4)$ ,  $B'(4, 1)$ ,  $C'(6, 4)$ . The checked  $y$ -coordinate of  $B'$  is  $4 - 3 = 1$ .

WORKED REASONING • A is the center, so  $A' = (4, 4)$ . B is 3 units east of A. A clockwise quarter turn changes east to south, so  $B'$  is 3 units south of A:  $(4, 4 - 3) = (4, 1)$ . C is 2 units north of A. Clockwise, north becomes east, so  $C'$  is 2 units east of A:  $(6, 4)$ . The right angle is still at A, and the sides are still 3 and 2 units long.

WATCH FOR • Students often rotate about the origin  $(0, 0)$  or about the middle of the shape instead of the marked point. Always start from the stated center. A point on the center does not move.

TRANSFER •  $B'$  is  $(4, 7)$  and  $C'$  is  $(2, 4)$ . Counterclockwise, east becomes north and north becomes west.

## 10. Say It Exactly

LEVEL 2

Design Y is the hidden design.

WORKED REASONING • The vague words fit W, X, and Y, because next to and in a corner can each mean several squares. Clue 1 keeps W, X, and Y, because each has its third square right above its second; Z goes down instead. Clue 2 keeps W and Y, because X's second square is to the left of the middle. Clue 3 keeps only Y, because W's corner square is  $(5, 5)$ . The only design that passes every exact clue is Y:  $(3, 3)$ ,  $(4, 3)$ ,  $(4, 4)$ ,  $(1, 1)$ .

WATCH FOR • Writers often think next to and in a corner are clear because they can see the picture. The reader cannot see it. Each vague word should become a direction plus a distance, or a coordinate.

TRANSFER •  $(1, 2)$ ,  $(2, 2)$ ,  $(2, 4)$ ,  $(5, 5)$ .

### 17. Turn Direction Equivalence

LEVEL 3

A single turn of 180 degrees clockwise (a half turn). The clockwise turns total  $270 + 90 = 360$  degrees, and  $360 - 180 = 180$ . The arrow ends pointing east.

**WORKED REASONING** • Add the clockwise turns:  $270 + 90 = 360$  degrees. Subtract the counterclockwise turn, which undoes part of that turning:  $360 - 180 = 180$  degrees clockwise. A half turn points the opposite way, so west becomes east. Step check: from west, 270 degrees clockwise goes W to N to E to S, so the arrow points south. Then 180 degrees counterclockwise makes it point north. Then 90 degrees clockwise makes it point east. Both methods give east.

**WATCH FOR** • Students may add all the degrees,  $270 + 180 + 90 = 540$ , and ignore direction. Counterclockwise turns undo clockwise turns, so they must be subtracted.

**TRANSFER** • Both end pointing north. 270 degrees clockwise goes E to S to W to N. 90 degrees counterclockwise goes E to N.

### 18. Diagonal Mirrors

LEVEL 3

(4, 1) is the image of A.

**WORKED REASONING** • Clue 1 ( $x$  greater than  $y$ ) keeps (4, 1), (3, 2), and (6, 3). Clue 2 ( $x + y = 5$ ) keeps (4, 1), (1, 4), and (3, 2). Clue 3 ( $x - y = 3$ ) keeps (4, 1) and (6, 3). Only (4, 1) passes all three clues. Step check: from A(1, 4), go 1 right and 1 down three times: (2, 3), (3, 2), (4, 1). The path has 3 steps, so its halfway point comes after 1.5 steps. That point is (2.5, 2.5), halfway between (2, 3) and (3, 2), and it lies on the mirror. So A and its image are the same distance from the mirror.

**WATCH FOR** • Students used to vertical mirrors may slide the point straight sideways to the mirror and beyond. For a diagonal mirror, the path must cross the mirror at a right angle, which is a diagonal path on the grid.

**TRANSFER** • (6, 2). The path takes 4 steps: (3, 5), (4, 4), (5, 3), (6, 2). Its halfway point, (4, 4), is on the mirror. For this mirror, the two coordinates trade places.