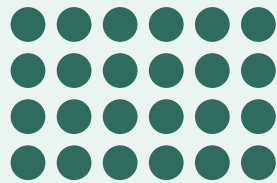
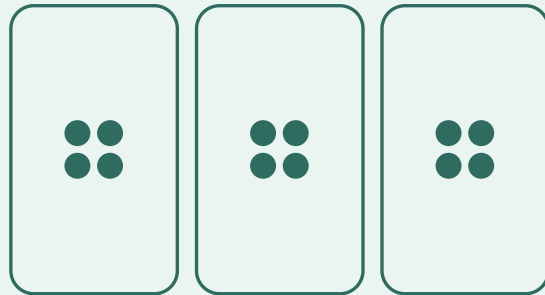


Multiplication Understanding: From Arrays to Partial Products and Beyond

6 complete activities from Multiplication Understanding, each with a worked answer.



4 rows of 6



3 groups of 4

HOW TO USE

1. Read the situation and the evidence first.
2. Show your thinking in the work boxes, not only an answer.
3. Explain how you know your answer makes sense.
4. Try the new problem at the bottom of the page.

6 activities • 3 answer pages • From the 36-activity collection

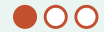
10 US Letter pages • Print at 100% / Actual Size

Original Bobotemp educational printable • Fictional names, places and prices

Name _____ Date _____

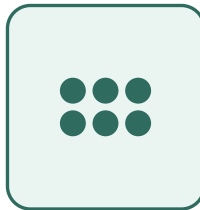
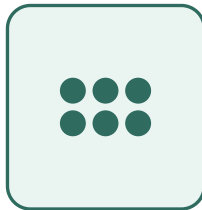
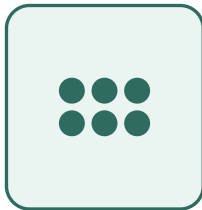
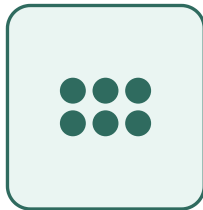
Array Builders

LEVEL 1



Read each story. Draw an array for each equal-groups story, and write an equation for every story.

Story A: A teacher sets out 4 trays with 6 markers on each tray. Story B: A bulletin board has 4 rows of photos with 6 photos in each row. Story C: Maya puts 6 stickers on a page, then adds 4 more stickers. Draw an array for each story that is about equal groups. Decide which story does NOT match 4×6 , and write the equation that does match it.



Story A: 4 trays, 6 markers on each tray

EVIDENCE TO USE

- Rows go across. Columns go down.
- 4 rows of 6 = 4×6
- Equal groups: every group has the same number

1. Draw a 4-by-6 array and circle the 4 equal groups.

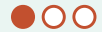
2. Write the multiplication equation for Stories A and B.

3. Tell which story does not match 4×6 and write its equation.

TRY IT • A shelf holds 3 rows of books with 7 books in each row. Write the equation and find the total.

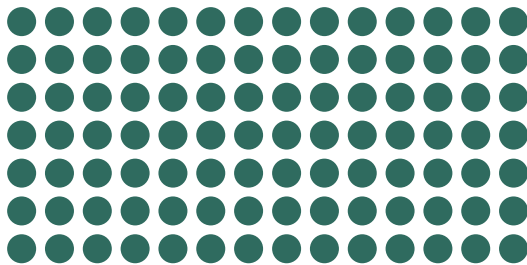
Name _____ Date _____

LEVEL 1



Cut the array into two friendly parts, find each part, and add.

A 7-by-14 array of tiles is too big to count quickly. Draw a line that cuts it into a 7-by-10 part and a 7-by-4 part. Find the number of tiles in each part, then add. Answer this question: Why does the total stay the same when you split the array?



7 rows, 14 columns

EVIDENCE TO USE

- 7 rows, 14 columns
- $14 = 10 + 4$
- $7 \times 14 = 7 \times 10 + 7 \times 4$

1. Find the number of tiles in the 7-by-10 part.

[illegible]

2. Find the number of tiles in the 7-by-4 part, then add the two parts.

[illegible]

3. Explain why splitting does not change the total.

TRY IT • Split a 6-by-13 array into two parts and find the total.

Name _____ Date _____

LEVEL 2



Build every rectangle you can with the tiles. Then list and count the factors.

Build every rectangle that can be made with exactly 36 square tiles, and record each factor pair. Then list all the factors of 36 in order. Explain why 36 has 5 factor pairs but only 9 factors, not 10.

EVIDENCE TO USE

- 36 square tiles
- Each factor pair makes a rectangle.
- A square is a rectangle too.

1. Record every factor pair of 36.

2. List all the factors of 36 in order.

A blank sheet of graph paper with a grid pattern. The grid consists of 10 columns and 10 rows of squares. The paper has rounded corners and a light gray border.

3. Explain why the 6-by-6 square adds only one factor.

[illegible]

TRY IT • How many factors does 49 have?

Name _____ Date _____

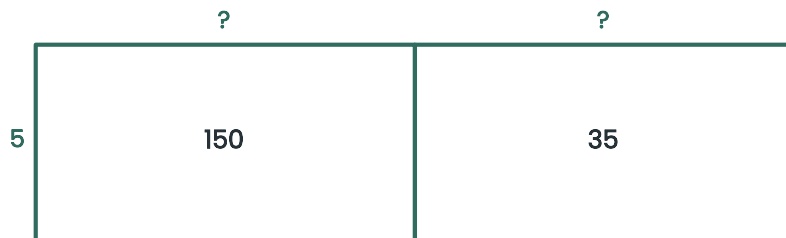
LEVEL 2



Work backward from the areas to find the side lengths.

An area model (not to scale) has one row split into two regions. The rectangle is 5 units tall. The left region has an area of 150, and the right region has an area of 35. Find the width of each region, the width of the whole rectangle, and the total area. Write the multiplication equation that the model shows.

NOT TO SCALE



EVIDENCE TO USE

- Height: 5
- Left region area: 150
- Right region area: 35
- Not to scale

1. Find the width of each region.

[illegible]

2. Write the two factors of the whole rectangle.

3. Find the total area and write the equation.

TRY IT • An area model is 8 units tall, and its two regions have areas of 320 and 56. What multiplication does it show?

Name _____ Date _____

LEVEL 3



The edge labels are missing. Use the partial areas to rebuild them.

A 2-by-2 area model lost its edge labels. The four partial areas are 1,200 (top left), 240 (top right), 210 (bottom left), and 42 (bottom right). The rows stand for the tens and ones of one two-digit factor. The columns stand for the tens and ones of the other. Find the four edge labels, the two factors, and the product. Check every region against your labels.

EDGE LABELS LOST

	?	?
?	1,200	240
?	210	42

EVIDENCE TO USE

- Top left: 1,200
- Top right: 240
- Bottom left: 210
- Bottom right: 42

1. Find edge labels that make both bottom regions, 210 and 42, work.

2. Use the top regions to find the last label, then write the two factors.

[illegible]

3. Add the partial areas and check with a different method.

TRY IT • The partial areas are 1,000 (top left), 60 (top right), 300 (bottom left), and 18 (bottom right). Find the factors and the product.

Name _____ Date _____

LEVEL 3



Find the largest square tile that covers the rectangle exactly.

A rectangular floor plan is 12 feet by 18 feet. It must be covered exactly with equal square tiles: whole tiles only, with no gaps or overlaps. Find the largest square tile that works and how many of those tiles are needed. Explain why a 4-foot tile fails and why a 9-foot tile fails.

EVIDENCE TO USE

- Floor: 12 ft by 18 ft
- A tile side must fit both sides exactly.
- Factors of 12: 1, 2, 3, 4, 6, 12
- Factors of 18: 1, 2, 3, 6, 9, 18

1. Circle the common factors of 12 and 18.

[illegible]

2. Find how many 6-foot tiles fit along each side, then find the total.

A blank sheet of graph paper with a grid pattern. The grid consists of 10 columns and 10 rows of squares. The paper has rounded corners and a light gray background.

3. Explain why 4-foot and 9-foot tiles do not work.

TRY IT • Find the largest square tile that exactly covers a 15-by-40 rectangle, and find how many tiles are needed.

1. Array Builders

LEVEL 1

Stories A and B match $4 \times 6 = 24$. Story C does not match; it is $6 + 4 = 10$.

WORKED REASONING • Story A has 4 equal groups of 6 markers: $6 + 6 + 6 + 6 = 24$, so $4 \times 6 = 24$. Story B has 4 rows with 6 in each row. That is a 4-by-6 array with 24 photos. Each row of the array is one equal group, so the array and the trays show the same product. Story C joins a group of 6 and a group of 4. The groups are not equal, so the story is addition: $6 + 4 = 10$.

WATCH FOR • Students may see the numbers 4 and 6 and multiply automatically. Multiplication needs equal groups. Story C has two unequal parts that are joined, so it is addition.

TRANSFER • $3 \times 7 = 21$ books.

2. Split the Array

LEVEL 1

$7 \times 14 = 70 + 28 = 98$ tiles.

WORKED REASONING • Split the 14 columns into 10 columns and 4 columns. Left part: $7 \times 10 = 70$. Right part: $7 \times 4 = 28$. Add: $70 + 28 = 98$. Every tile is in exactly one part, so no tile is lost or counted twice. That is why the total stays 98.

WATCH FOR • Students may find 7×10 and then just add 4, getting 74. Each of the 4 columns has 7 tiles, so the second part is $7 \times 4 = 28$, not 4.

TRANSFER • $6 \times 10 + 6 \times 3 = 60 + 18 = 78$.

13. Factor Pair Hunt

LEVEL 2

36 has 5 factor pairs and 9 factors: 1, 2, 3, 4, 6, 9, 12, 18, 36.

WORKED REASONING • Test 1 through 6 rows: 1×36 , 2×18 , 3×12 , 4×9 , and 6×6 . Five rows do not work, because 36 tiles do not split into 5 equal rows. After 6, the pairs repeat. The four pairs with two different numbers give $4 \times 2 = 8$ factors. The pair 6×6 uses the same number twice, so it adds only 1 factor: $8 + 1 = 9$ factors.

WATCH FOR • Students may double the number of pairs and say 36 has 10 factors, which counts 6 twice. Each factor is listed only once, even when it appears twice in a pair.

TRANSFER • 49 has 2 factor pairs (1×49 and 7×7), so it has 3 factors: 1, 7, and 49.

14. Missing Dimensions

LEVEL 2

The widths are 30 and 7, so the model shows $5 \times 37 = 150 + 35 = 185$.

WORKED REASONING • Left region: $5 \times ? = 150$, and $150 \div 5 = 30$, so its width is 30. Right region: $5 \times ? = 35$, and $35 \div 5 = 7$, so its width is 7. Whole width: $30 + 7 = 37$. Total area: $150 + 35 = 185$. So $5 \times 37 = 185$. The 150 is 5×3 tens, and the 35 is 5×7 ones.

WATCH FOR • Students may add the areas and say the width is $150 + 35$. The numbers inside the regions are areas, not lengths. Each width comes from dividing a region's area by the shared height.

TRANSFER • The widths are 40 and 7, so the model shows $8 \times 47 = 320 + 56 = 376$.

25. Reverse Area Model

LEVEL 3

The factors are 47 and 36, and $47 \times 36 = 1,200 + 240 + 210 + 42 = 1,692$.

WORKED REASONING • The bottom row holds 210 and 42. The 42 is ones x ones, so try 7×6 . Then $7 \times 30 = 210$ fits, which makes the column labels 30 and 6. Top row: $1,200 \div 30 = 40$ and $240 \div 6 = 40$, so the top row label is 40. The row labels give $40 + 7 = 47$, and the column labels give $30 + 6 = 36$. Total: $1,200 + 240 + 210 + 42 = 1,692$. Check with compensation: $47 \times 36 = 47 \times 40 - 47 \times 4 = 1,880 - 188 = 1,692$.

WATCH FOR • Students may pick a factor pair for 42, such as 2×21 or 6×7 , without checking the other regions. Every edge label must work for both regions it touches.

TRANSFER • The rows are 20 and 6, and the columns are 50 and 3, so the model shows $26 \times 53 = 1,378$.

26. Largest Square Tiles

LEVEL 3

The largest tile is 6 feet by 6 feet, and $2 \times 3 = 6$ tiles are needed.

WORKED REASONING • A tile side must divide both 12 and 18. The common factors are 1, 2, 3, and 6, and the greatest is 6. Along the 12-ft side: $12 \div 6 = 2$ tiles. Along the 18-ft side: $18 \div 6 = 3$ tiles. Total: $2 \times 3 = 6$ tiles. Area check: 6 tiles x 36 square feet = 216 square feet, and $12 \times 18 = 216$. A 4-ft tile fits the 12-ft side, but 4 tiles cover only 16 ft of the 18-ft side, which leaves 2 ft. A 9-ft tile fits 18 but not 12.

WATCH FOR • Students may choose the largest factor of only one side, or think that needing more tiles means using bigger tiles. The tile side must divide both sides, and bigger tiles mean fewer tiles.

TRANSFER • Use 5-by-5 tiles. You need $3 \times 8 = 24$ tiles.