

Math Talk Reasoning Routines: Strategy, Compare, and Context Talks

6 complete activities from Math Talk Reasoning Routines, each with a worked answer.

Notice

What do you see?

Wonder

What could it mean?

Justify

How do you know?

FOR THE TEACHER

1. Choose the page that matches the skill you are working on.
2. Let students work, then listen to how they explain it.
3. Use the answer key's "Watch for" note to plan your next question.
4. Use the Try It task to check that the idea transfers.

6 activities • 3 answer pages • From the 48-activity collection

10 US Letter pages • Print at 100% / Actual Size

Original Bobotemp educational printable • Fictional names, places and prices

Name _____ Date _____

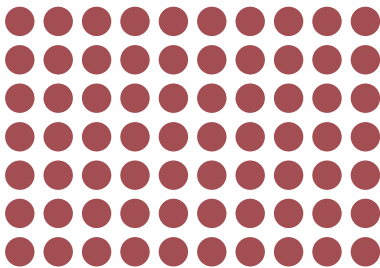
LEVEL 1



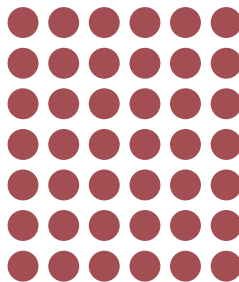
Think quietly first. Then share your strategy with a partner using a sentence starter.

How can you find 7×16 without writing the standard algorithm? The array has 7 rows of 16 dots. A line splits every row into 10 dots and 6 dots. Use the two smaller rectangles to find the product. Then explain your thinking to a partner.

7 ROWS OF 16 DOTS, SPLIT 10 | 6



Left: 7×10



Right: 7×6

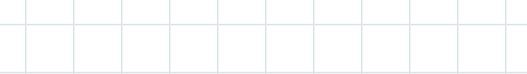
EVIDENCE TO USE

- 7 rows of 16 dots
- Split line: 10 dots | 6 dots
- Left rectangle: 7×10
- Right rectangle: 7×6
- Starter: I split ___ into ___ because ___.

1. Find the product for each smaller rectangle.

[illegible]

2. Add the partial products and write one equation for the whole array.

A large rectangular area filled with a light gray grid, intended for drawing a picture. The grid consists of 10 columns and 10 rows of squares.

3. Sketch the array, shade where 42 lives, and tell why.

TRY IT • Use an array split to find 6×14 .

Name _____ Date _____

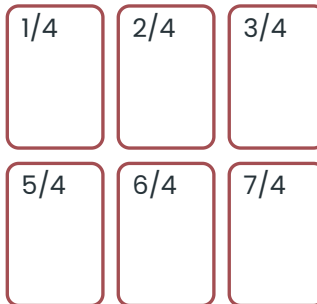
LEVEL 1



Read one clue at a time. Cross out cards that do not fit, then tell a partner how you know.

A number line goes from 0 to 2 and is partitioned into fourths. Six fraction cards show possible spots for a mystery point: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$. Use every clue to find the mystery point. Then mark it on the line and explain which tick mark it is on.

CROSS OUT



- Number line: 0 to 2, marked in fourths
- Cards: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{5}{4}$, $\frac{6}{4}$, $\frac{7}{4}$
- Clue 1: It is more than $\frac{1}{2}$.
- Clue 2: It is less than $1\frac{1}{2}$.
- Clue 3: It is not between 0 and 1.

1. Cross out the cards that fail each clue.

[illegible]

3. Tell how many fourths it is from 0 and from 1.

[illegible]

TRY IT • On a 0 to 2 number line marked in thirds, which tick mark shows $\frac{4}{3}$?

Name _____ Date _____

LEVEL 2



Try both students' strategies. Then compare them with a partner and decide when each one is efficient.

Ari says, 'To find 8×25 , I halve 8 and double 25 until the problem gets easy.' Bea says, 'To find 9×19 , I think of 9×20 and then fix it.' Try both strategies. Which strategy fits which problem better, and why? Give 9×19 as your final checked answer.

EVIDENCE TO USE

- Ari: $8 \times 25 \rightarrow 4 \times 50 \rightarrow 2 \times 100$
- Bea: $9 \times 19 \rightarrow$ think 9×20 , then adjust
- 9×20 means 9 rows of 20
- 9×19 means 9 rows of 19

1. Finish Ari's chain and write the product of 8×25 .

2. Explain why Bea subtracts 9, not 1, after finding 9×20 . 3. Choose a strategy for 4×49 and tell why.

A blank sheet of graph paper with a grid pattern. The grid consists of 10 columns and 10 rows of squares. The lines are light gray, and the background is white. There are no margins or other markings on the page.[illegible]

TRY IT • Use compensation to find 6×29 .

Name _____ Date _____

LEVEL 2



Compare two incorrect placements, then use a number line to find the correct one.

Jai placed $\frac{5}{3}$ at the 5 on a ruler. Lena placed $\frac{5}{3}$ between 0 and 1. Use a number line from 0 to 2 partitioned into thirds to find where $\frac{5}{3}$ really belongs. How many thirds past 1 is it? Write it as a mixed number. Then convince a partner that Jai and Lena made different mistakes.

0 TO 2, EACH WHOLE IN THIRDS



EVIDENCE TO USE

- Number line: 0 to 2, each whole cut into 3 equal parts
- $3/3 = 1$
- $6/3 = 2$
- Jai: $5/3$ is at 5
- Lena: $5/3$ is between 0 and 1

1. Count jumps of $\frac{1}{3}$ from 0 and label each tick mark.

2. Tell how many thirds past 1 the point $\frac{5}{3}$ is, and write $\frac{5}{3}$ as a mixed number.

A blank sheet of graph paper with a grid of squares. The grid consists of 10 columns and 10 rows of squares. The lines are thin and gray, forming a uniform pattern across the page. There are no margins or additional markings on the paper.

3. Explain each student's mistake in one sentence.

TRY IT • On a 0 to 2 number line marked in fourths, where is $\frac{7}{4}$?

Name _____ Date _____

LEVEL 3



Use the clues to find the mystery card. Then build a counterexample to test a classmate's rule.

Dev claims, 'You can make an equivalent fraction by adding the same number to the top and bottom.' Six cards are on the board: $\frac{4}{6}$, $\frac{6}{10}$, $\frac{8}{10}$, $\frac{9}{15}$, $\frac{12}{20}$, $\frac{5}{7}$. Use the clues to find the mystery card. Then apply Dev's rule to $\frac{3}{5}$ and prove his claim is false with a counterexample.

EVIDENCE TO USE

- Cards: $\frac{4}{6}$, $\frac{6}{10}$, $\frac{8}{10}$, $\frac{9}{15}$, $\frac{12}{20}$, $\frac{5}{7}$
- Clue 1: It is equivalent to $\frac{3}{5}$.
- Clue 2: Its denominator is odd.
- Clue 3: Its numerator is greater than 6.

1. List the cards that survive each clue, then name the mystery card.

2. Show the multiplication that turns $\frac{3}{5}$ into the mystery card.

A blank sheet of graph paper with a grid pattern. The grid consists of 10 columns and 8 rows of squares. The paper has rounded corners and a light gray border.

3. Use Dev's rule on $3/5$ and prove the result is not equivalent.

TRY IT • Apply Dev's rule to $\frac{2}{3}$ by adding 1 to the top and bottom. Is the new fraction equivalent to $\frac{2}{3}$? Prove it.

Name _____ Date _____

LEVEL 3



Compare fractions near 1 by thinking about the piece each one is missing. Then use that idea to solve the clues.

Which is greater, $\frac{7}{8}$ or $\frac{9}{10}$? Both are close to 1, so think about the piece each fraction is missing from one whole. Then use the same reasoning to find the mystery fraction among these cards: $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{9}{10}$, $\frac{4}{5}$, $\frac{2}{3}$.

EVIDENCE TO USE

- $\frac{7}{8}$ is missing $\frac{1}{8}$ of a whole
- $\frac{9}{10}$ is missing $\frac{1}{10}$ of a whole
- Clue 1: It is greater than $\frac{3}{4}$.
- Clue 2: It is closer to 1 than $\frac{5}{6}$ is.
- Clue 3: Its missing piece is smaller than $\frac{1}{9}$.

1. Compare $\frac{7}{8}$ and $\frac{9}{10}$ by comparing their missing pieces.

[illegible]

2. List the cards that survive each clue and name the mystery fraction.

[illegible]

3. Explain why a smaller missing piece means a greater fraction.

[illegible]

TRY IT • Which is greater, $\frac{4}{5}$ or $\frac{7}{9}$? Hint: the missing pieces are $\frac{1}{5}$ and $\frac{2}{9}$.

1. Split the Array

LEVEL 1

$7 \times 16 = 112$, because $7 \times 10 = 70$, $7 \times 6 = 42$, and $70 + 42 = 112$.

WORKED REASONING • Break 16 into $10 + 6$. The left rectangle is $7 \times 10 = 70$. The right rectangle is $7 \times 6 = 42$. The two rectangles make the whole array, so $7 \times 16 = 70 + 42 = 112$. Other valid splits also work. Splitting 7 into $5 + 2$ gives $5 \times 16 + 2 \times 16 = 80 + 32 = 112$. Splitting 16 into $8 + 8$ gives $56 + 56 = 112$. Accept any split where every part is multiplied by the other factor.

WATCH FOR • A student may write $7 \times 10 + 6 = 76$, multiplying only the 10. The array shows 7 rows in the 6-dot part too. That part is $7 \times 6 = 42$, not 6. Both parts of a split must be multiplied by 7.

TRANSFER • $6 \times 14 = 6 \times 10 + 6 \times 4 = 60 + 24 = 84$.

2. Mystery Point on the Line

LEVEL 1

The mystery point is $5/4$.

WORKED REASONING • Clue 1: $1/2 = 2/4$, so $3/4$, $5/4$, $6/4$, and $7/4$ remain. Clue 2: $1 \frac{1}{2} = 6/4$, so $1/4$, $2/4$, $3/4$, and $5/4$ remain. $6/4$ is exactly $1 \frac{1}{2}$, so it is not less. Clue 3: $5/4$, $6/4$, and $7/4$ are not between 0 and 1. Only $5/4$ passes all three clues. It is 5 jumps of $1/4$ from 0, which is 1 jump past 1.

WATCH FOR • Some students think a fraction can never be more than 1, so they search only between 0 and 1. Counting jumps of $1/4$ from 0 shows that $4/4$ lands on 1. The next jump lands on $5/4$, past 1.

TRANSFER • $4/3$ is the first tick mark after 1. It is 4 jumps of $1/3$ from 0, so $4/3 = 1 \frac{1}{3}$.

17. Compensation and Doubling

LEVEL 2

$9 \times 19 = 171$, because $9 \times 20 = 180$ and $180 - 9 = 171$. Ari's chain also gives $8 \times 25 = 200$.

WORKED REASONING • Ari's strategy: halving one factor and doubling the other keeps the product the same, so $8 \times 25 = 4 \times 50 = 2 \times 100 = 200$. Bea's strategy: 19 is 1 less than 20, so 9×19 is 9 rows that each have 1 fewer than 20. First, $9 \times 20 = 180$. Then remove 1 from each of the 9 rows: $180 - 9 = 171$. Doubling and halving works well when a factor can become 10 or 100. Compensation works well when a factor is close to a multiple of 10. For 4×49 , compensation gives $200 - 4 = 196$.

WATCH FOR • Students often compute $9 \times 20 - 1 = 179$. Subtracting 1 removes only one dot, but each of the 9 rows lost one, so 9 must be subtracted. To repair it, draw 9 rows of 20 and cross out one dot in each row.

TRANSFER • $6 \times 29 = 6 \times 30 - 6 = 180 - 6 = 174$.

18. Beyond One

LEVEL 2

$5/3$ is 2 thirds past 1 ($5 - 3 = 2$), so $5/3 = 1 \frac{2}{3}$. It lies between 1 and 2.

WORKED REASONING • Each whole has 3 thirds, so $1 = 3/3$ and $2 = 6/3$. Count the jumps: $1/3, 2/3, 3/3$ (which is 1), $4/3, 5/3$. $5/3$ is $5 - 3 = 2$ thirds past 1, so $5/3 = 1 \frac{2}{3}$. It is one third less than 2. Jai read the numerator as a whole-number location. Lena assumed the point must be less than 1 because it is a fraction.

WATCH FOR • Two errors appear here. One is treating the numerator as the location, which puts $5/3$ at 5. The other is believing all fractions are less than 1. The labeled number line repairs both, because the third tick past 0 is 1 and counting continues past it.

TRANSFER • $7/4$ is 3 fourths past 1, so $7/4 = 1 \frac{3}{4}$. It is one fourth less than 2.

33. Name Families and the Adding Trap

LEVEL 3

The mystery card is $9/15$, because $3/5 = (3 \times 3)/(5 \times 3) = 9/15$.

WORKED REASONING • Clue 1: multiplying $3/5$ by $2/2$, $3/3$, and $4/4$ gives $6/10$, $9/15$, and $12/20$. Clue 2: the odd denominators are 15 and 7, which leaves $9/15$ and $5/7$. Clue 3: the numerators greater than 6 are 8, 9, and 12. Only $9/15$ survives all three clues. Counterexample: Dev's rule turns $3/5$ into $4/6$ (add 1 to top and bottom). But $3/5 = 18/30$ and $4/6 = 20/30$, so they are not equal. Multiplying splits every piece into equal smaller pieces. Adding changes the amount.

WATCH FOR • The 'add the same number' rule feels like a pattern, but it changes the size of the amount. The cards $4/6$ and $5/7$ were built with Dev's rule from $3/5$. Renaming each pair with a common unit shows both are greater than $3/5$: in thirtieths, $4/6 = 20/30$ and $3/5 = 18/30$, and in thirty-fifths, $5/7 = 25/35$ and $3/5 = 21/35$. Lining up fraction strips shows the same thing.

TRANSFER • Dev's rule gives $3/4$. Since $2/3 = 8/12$ and $3/4 = 9/12$, they are not equivalent. $3/4$ is greater.

34. Distance from One

LEVEL 3

The mystery fraction is $9/10$, and $9/10$ is greater than $7/8$.

WORKED REASONING • $7/8$ is $1/8$ away from 1, and $9/10$ is $1/10$ away. Tenths are smaller pieces than eighths, so $9/10$ is closer to 1 and greater. Clue 1: every card is exactly one unit piece short of 1 ($3/4$ misses $1/4$, $5/6$ misses $1/6$, $7/8$ misses $1/8$, $9/10$ misses $1/10$, $4/5$ misses $1/5$, $2/3$ misses $1/3$). Cards missing less than $1/4$ are greater than $3/4$, so $5/6$, $7/8$, $9/10$, and $4/5$ survive. $3/4$ itself is out because it is equal to $3/4$, not greater. $2/3$ is out because it misses $1/3$, which is more than $1/4$. Clue 2: $5/6$ misses $1/6$, and only $7/8$ and $9/10$ miss less. Clue 3: $1/10$ is smaller than $1/9$, but $1/8$ is not. Mystery fraction: $9/10$.

WATCH FOR • Students may say $7/8$ is greater because eighths are bigger pieces. Others call $7/8$ and $9/10$ equal because each is 'one piece from a whole.' Bigger pieces make the missing part bigger, so $7/8$ is farther from 1. The fraction with the smaller missing piece is greater.

TRANSFER • $4/5$ is greater. $1/5 = 2/10$, and $2/9$ is more than $2/10$ because ninths are bigger than tenths. So $7/9$ is missing more, and $4/5$ is closer to 1.