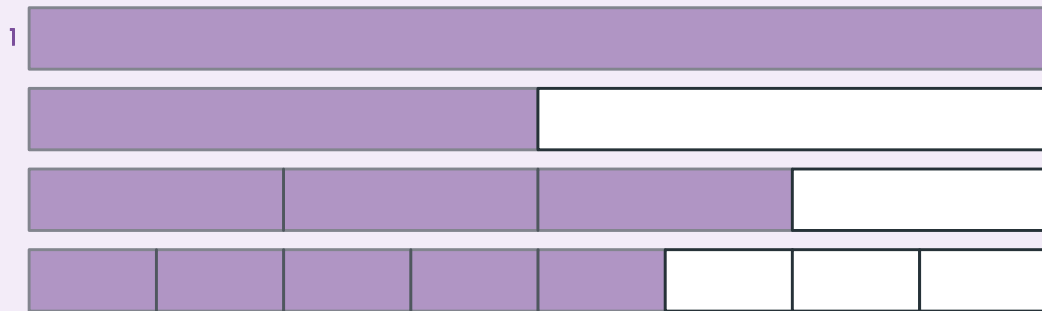


Fraction Sense Lab: Units, Comparisons, and Operations

6 complete activities from Fraction Sense Lab, each with a worked answer.



HOW TO USE

1. Read the situation and the evidence first.
2. Show your thinking in the work boxes, not only an answer.
3. Explain how you know your answer makes sense.
4. Try the new problem at the bottom of the page.

6 activities • 4 answer pages • From the 36-activity collection

11 US Letter pages • Print at 100% / Actual Size

Original Bobotemp educational printable • Fictional names, places and prices

Name _____ Date _____

LEVEL 1



Fold or shade the strips. All strips are the same length. Use the clues to find the mystery piece.

Five paper strips are all the same length. Each strip is folded into equal parts: sixths, halves, eighths, thirds, and fourths. Order the unit fractions $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{3}$, and $\frac{1}{4}$ from greatest to least. Then solve the Mystery Piece riddle. I am one piece from one of these strips. Clue 1: I am smaller than $\frac{1}{3}$. Clue 2: I am larger than $\frac{1}{8}$. Clue 3: It takes more than 4 of me to make the whole strip. Which unit fraction am I?

FIVE STRIPS, SAME WHOLE

sixths	1/6	1/6	1/6	1/6	1/6	1/6	
halves	1/2			1/2			
eighths	1/8	1/8	1/8	1/8	1/8	1/8	1/8
thirds	1/3		1/3		1/3		
fourths	1/4		1/4		1/4		1/4

EVIDENCE TO USE

- All five strips are the same length (one whole).
- Compare the size of one part on each strip.
- Candidates: $\frac{1}{6}$, $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{3}$, $\frac{1}{4}$

1. Order the five unit fractions from greatest to least.

[illegible]

2. Cross out the candidates that fail each clue.

[illegible]

3. Explain why $\frac{1}{8}$ is smaller than $\frac{1}{3}$ even though $8 > 3$.

[illegible]

TRY IT • Two ribbons are the same length. TRY is cut into 5 equal pieces, and the other is cut into 10 equal pieces. Which piece is longer? How many of the shorter pieces match one longer piece?

Name _____ Date _____

LEVEL 1



Count intervals (jumps), not tick marks. Start counting at 0.

A number line goes from 0 to 2. Each whole is split into 4 equal intervals, so the tick marks are $\frac{1}{4}$ apart. Place $\frac{3}{4}$, $\frac{5}{4}$, and $1\frac{1}{2}$ on the line. For each point, count the jumps of $\frac{1}{4}$ from 0 and write the count next to the point. How many jumps of $\frac{1}{4}$ does it take to reach $1\frac{1}{2}$?

0 TO 2 IN FOURTHS



EVIDENCE TO USE

- The line runs from 0 to 2.
- Each interval is $1/4$.
- Points to place: $3/4, 5/4, 1\ 1/2$

1. Mark and label each point on the line.

[illegible]

2. Write how many $\frac{1}{4}$ jumps you counted for each point.

[illegible]

3. Explain why $\frac{5}{4}$ belongs to the right of 1.

TRY IT • A number line from 0 to 3 is marked in thirds. Where does $2\frac{1}{3}$ go? How many jumps of $\frac{1}{3}$ is it from 0?

Name _____ Date _____

Same Whole?

LEVEL 2



Find each area before you decide. Pay attention to the size of each whole.

Two rectangular garden plots are different sizes. Plot A has an area of 12 square yards. Plot B has an area of 24 square yards. Maya plants flowers on $\frac{1}{2}$ of Plot A. Theo plants flowers on $\frac{1}{3}$ of Plot B. Maya says, 'My part is bigger because $\frac{1}{2} > \frac{1}{3}$.' Find each flower area. Who planted more square yards, and by how much? As numbers, $\frac{1}{2} > \frac{1}{3}$. Explain when that tells you which flower part is bigger.

EVIDENCE TO USE

- Plot A: 12 square yards
- Plot B: 24 square yards
- Maya: $\frac{1}{2}$ of Plot A. Theo: $\frac{1}{3}$ of Plot B.

1. Draw each plot split into equal parts and find each flower area.

2. Find the difference between the two flower areas.

3. Explain when $\frac{1}{2} > \frac{1}{3}$ tells you which part is bigger, and why Maya's reasoning fails here.

TRY IT • Which is longer: $\frac{1}{4}$ of a 20-meter rope or $\frac{1}{2}$ of an 8-meter rope? By how much?

Name _____ Date _____

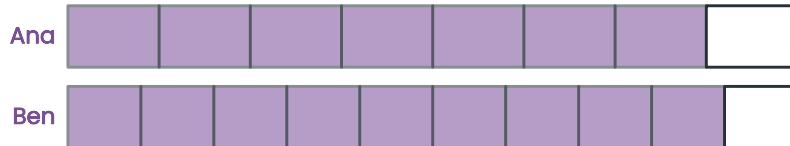
LEVEL 2



Compare how much is missing from each whole. Then check your result with a common denominator.

Two hikers walk the same 1-mile trail. Ana has walked $\frac{7}{8}$ of the trail. Ben has walked $\frac{9}{10}$ of the trail. Without using a common denominator, decide who is closer to the end by comparing the missing pieces. Then check with a common denominator: how much farther has the leader walked? Finally, use the missing-piece strategy to compare $\frac{5}{6}$ and $\frac{7}{8}$.

SAME 1-MILE TRAIL (SHADED = WALKED)



EVIDENCE TO USE

- Same trail: 1 mile
- Ana: $\frac{7}{8}$ of the trail. Ben: $\frac{9}{10}$ of the trail.
- Missing piece = distance from 1

1. Write each hiker's missing piece and tell who is ahead.

[illegible]

2. Rename both fractions in fortieths and find the difference.

[illegible]

3. Compare $\frac{5}{6}$ and $\frac{7}{8}$ using missing pieces.

TRY IT • Use missing pieces to compare $\frac{11}{12}$ and $\frac{14}{15}$.

Name _____ Date _____

LEVEL 3



Zoom in on a short segment of the number line. Re-partition it until every point lands on a tick.

A number line is marked only at $\frac{1}{2}$ and $\frac{2}{3}$. Zoom in on the space between those two points. You will place $\frac{7}{12}$ and $\frac{5}{8}$ in that space. First, rename $\frac{1}{2}$ and $\frac{2}{3}$ as twelfths and locate $\frac{7}{12}$. Next, explain why $\frac{5}{8}$ does not land on a twelfth tick, and re-partition the segment into twenty-fourths. What is $\frac{5}{8}$ in twenty-fourths? Which point is farther to the right?

RE-PARTITION IN TWENTY-FOURTHS



1. Rename $\frac{1}{2}$, $\frac{7}{12}$, and $\frac{2}{3}$ in twelfths, and then in twenty-fourths.

[illegible]

2. Write $\frac{5}{8}$ in twenty-fourths and mark both new points.

[illegible]

3. Explain which point is farther to the right and by how much.

TRY IT • Place $\frac{3}{5}$ between $\frac{1}{2}$ and $\frac{2}{3}$. What unit works, and where does $\frac{3}{5}$ go?

Name _____ Date _____

Impostor Equivalences

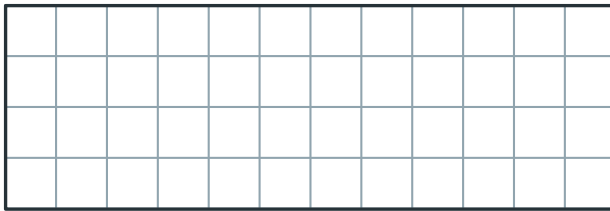
LEVEL 3



Test the claim with common denominators. Then explain which change keeps a fraction equivalent.

Jordan claims: 'If you add the same number to the top and bottom of a fraction, you get an equivalent fraction. So $2/3 = 3/4$ and $4/6 = 5/7$.' Test the claim. Rename $2/3$ and $3/4$ as twelfths. Rename $4/6$ and $5/7$ as forty-seconds. By how many forty-seconds do $4/6$ and $5/7$ differ? Then use an area model to explain which operation does keep a fraction equivalent.

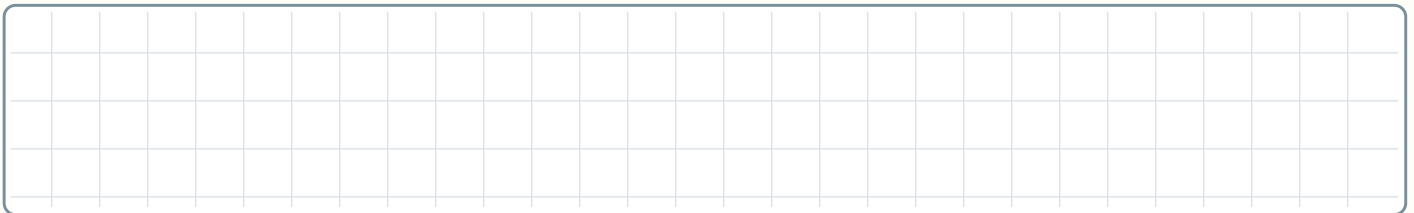
AREA MODEL SPACE



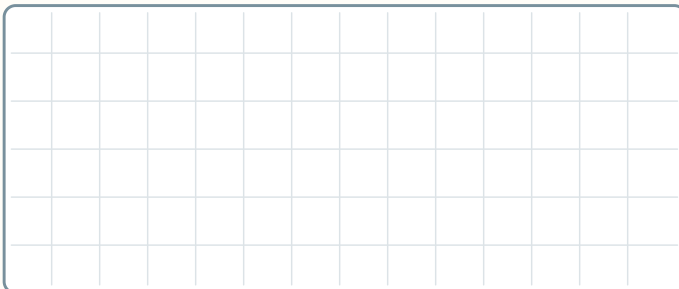
EVIDENCE TO USE

- Claim 1: $2/3 = 3/4$
- Claim 2: $4/6 = 5/7$
- Twelfths fit thirds and fourths.
- Forty-seconds fit sixths and sevenths.

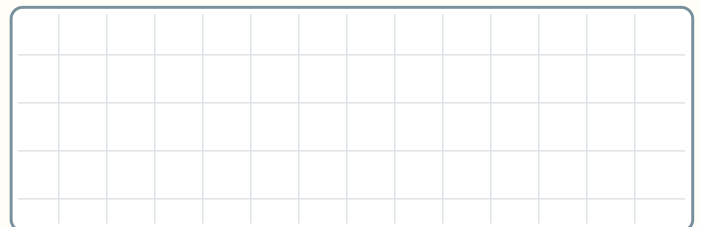
1. Rename both pairs and decide whether each pair is equal.



2. Find the difference between $4/6$ and $5/7$ in forty-seconds.



3. Use an area model on the grid to show which change to both terms keeps a fraction equivalent, and why adding 1 to both terms does not.



TRY IT • Is $3/5 = 6/8$? Is $3/5 = 6/10$? Explain each answer.

1. Fair Share Folding

LEVEL 1

Order: $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{6} > \frac{1}{8}$. The mystery piece is $\frac{1}{6}$.

WORKED REASONING • The wholes are the same size, so the strip cut into more parts has smaller parts: $\frac{1}{2} > \frac{1}{3} > \frac{1}{4} > \frac{1}{6} > \frac{1}{8}$. Clue 1 (smaller than $\frac{1}{3}$) leaves $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$. Of those, Clue 2 (larger than $\frac{1}{8}$) leaves $\frac{1}{4}$ and $\frac{1}{6}$. Clue 3 (more than 4 copies make the whole) leaves only $\frac{1}{6}$: 6 sixths make the whole, but only 4 fourths. So the mystery piece is $\frac{1}{6}$.

WATCH FOR • Students who use whole-number thinking may say $\frac{1}{8} > \frac{1}{3}$ because $8 > 3$. The denominator tells how many equal parts the whole is cut into. When the whole stays the same, a larger denominator means smaller parts.

TRANSFER • A fifth is longer. Two tenths match one fifth, because $\frac{1}{5} = \frac{2}{10}$.

2. Number Line Warm-Up

LEVEL 1

It takes 6 jumps of $\frac{1}{4}$ to reach $1 \frac{1}{2}$, so $1 \frac{1}{2} = \frac{6}{4}$ (also $\frac{3}{2}$). $\frac{3}{4}$ is 3 jumps from 0, and $\frac{5}{4}$ is 5 jumps from 0.

WORKED REASONING • Each whole holds 4 jumps of $\frac{1}{4}$. $\frac{3}{4}$ is 3 jumps, one tick before 1. $\frac{5}{4}$ is 5 jumps: 4 jumps reach 1, and 1 more jump makes $\frac{5}{4} = 1 \frac{1}{4}$. For $1 \frac{1}{2}$, it takes 4 jumps to reach 1, then 2 more jumps for $\frac{1}{2}$ (because $\frac{2}{4} = \frac{1}{2}$), so 6 jumps in all: $1 \frac{1}{2} = \frac{6}{4}$. Check: $1 \frac{1}{2} \times 4 = 6$.

WATCH FOR • Some students count tick marks instead of intervals and call the 0 tick '1'. This lands every point one fourth too far to the left. Others think $\frac{5}{4}$ cannot go on a number line because its numerator is larger than its denominator.

TRANSFER • It is 7 jumps of $\frac{1}{3}$ from 0, one tick past 2, because $2 \frac{1}{3} = \frac{7}{3}$.

13. Same Whole?

LEVEL 2

Maya's flower area is 6 square yards, and Theo's is 8 square yards. Theo planted 2 square yards more ($8 - 6 = 2$).

WORKED REASONING • To find $\frac{1}{2}$ of 12, split 12 into 2 equal parts of 6 each. Maya planted 6 square yards. To find $\frac{1}{3}$ of 24, split 24 into 3 equal parts of 8 each. Theo planted 8 square yards. Since $8 - 6 = 2$, Theo planted 2 square yards more. As numbers, $\frac{1}{2} > \frac{1}{3}$ is always true, so half of a whole is more than a third of the same whole. Here the wholes are different sizes (24 is twice 12), so the fractions alone cannot tell which part is bigger; find each part first.

WATCH FOR • Some students compare fractions without checking the wholes. As numbers, $\frac{1}{2} > \frac{1}{3}$ is always true, but it tells you which part is bigger only when both parts come from the same size whole. With different wholes, find the size of each part before comparing.

TRANSFER • $\frac{1}{4}$ of the 20-meter rope is 5 m, and $\frac{1}{2}$ of the 8-meter rope is 4 m. The first piece is longer by 1 m.

14. Missing Piece Comparisons

LEVEL 2

Ben is ahead. $\frac{9}{10} - \frac{7}{8} = \frac{36}{40} - \frac{35}{40} = \frac{1}{40}$ mile (1 fortieth). Also, $\frac{5}{6} < \frac{7}{8}$.

WORKED REASONING • Ana is missing $\frac{1}{8}$ of the trail, and Ben is missing $\frac{1}{10}$. A tenth is smaller than an eighth of the same whole, so Ben has less left to walk and has walked farther: $\frac{9}{10} > \frac{7}{8}$. Check with fortieths: $\frac{7}{8} = \frac{35}{40}$ and $\frac{9}{10} = \frac{36}{40}$, so $\frac{36}{40} - \frac{35}{40} = \frac{1}{40}$ mile. For $\frac{5}{6}$ and $\frac{7}{8}$, the missing pieces are $\frac{1}{6}$ and $\frac{1}{8}$. Since $\frac{1}{8} < \frac{1}{6}$, $\frac{7}{8}$ is closer to 1, so $\frac{5}{6} < \frac{7}{8}$.

WATCH FOR • Some students think $\frac{7}{8}$ and $\frac{9}{10}$ are equal because each is 'one piece away' from a whole. The missing pieces are different sizes, so the fractions are different.

TRANSFER • $\frac{14}{15} > \frac{11}{12}$. The missing pieces are $\frac{1}{15}$ and $\frac{1}{12}$, and $\frac{1}{15}$ is smaller, so $\frac{14}{15}$ is closer to 1.

25. Zoom Lines

LEVEL 3

$5/8 = 15/24$ and $7/12 = 14/24$. Both lie between $1/2 = 12/24$ and $2/3 = 16/24$. $5/8$ is farther to the right, by $1/24$.

TWELFTHS: $5/8$ FALLS BETWEEN TICKS



TWENTY-FOURTHS: EVERY POINT ON A TICK



WORKED REASONING • In twelfths, $1/2 = 6/12$ and $2/3 = 8/12$, so $7/12$ is the single tick halfway between them. $5/8$ is not a whole number of twelfths: $5 \times 12 \div 8 = 7.5$, so $5/8$ is $7 \frac{1}{2}$ twelfths and would sit halfway between $7/12$ and $8/12$. (Some eighths do land on twelfths, such as $6/8 = 9/12$, but $5/8$ does not.) Use a unit that both 8 and 12 divide into evenly: twenty-fourths. Split each twelfth in half: $1/2 = 12/24$, $7/12 = 14/24$, and $2/3 = 16/24$. Then $5/8 = (5 \times 3)/(8 \times 3) = 15/24$. The segment from $12/24$ to $16/24$ has 4 intervals. $7/12$ is two intervals to the right of $1/2$, and $5/8$ is three intervals to the right, so $5/8$ is $1/24$ farther right than $7/12$.

WATCH FOR • Some students guess where $5/8$ goes, putting it outside the segment or on top of $7/12$ because the two fractions look close. Every tick must stand for the same unit, so rename all the values in one unit to find their exact positions.

TRANSFER • Use thirtieths. $1/2 = 15/30$, $2/3 = 20/30$, and $3/5 = 18/30$, so $3/5$ is three thirtieth intervals to the right of $1/2$.

26. Impostor Equivalences

LEVEL 3

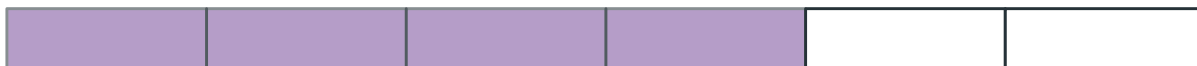
Jordan is wrong. $\frac{2}{3} = \frac{8}{12}$, which is less than $\frac{3}{4} = \frac{9}{12}$. Also, $\frac{4}{6} = \frac{28}{42}$ and $\frac{5}{7} = \frac{30}{42}$, so they differ by 2 forty-seconds ($\frac{2}{42} = \frac{1}{21}$).

SPLIT EACH THIRD IN TWO: $\frac{2}{3} = \frac{4}{6}$, SAME SHADED AREA

thirds



sixths



WORKED REASONING • $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$, so $\frac{3}{4}$ is $\frac{1}{12}$ more than $\frac{2}{3}$. Next, $\frac{4}{6} = \frac{28}{42}$ (multiply both terms by 7) and $\frac{5}{7} = \frac{30}{42}$ (multiply both terms by 6). Since $\frac{30}{42} - \frac{28}{42} = \frac{2}{42}$, the two fractions are not equal. Adding 1 to both terms adds one more part to the whole and one more shaded part, which generally changes the amount (it only keeps the value for fractions equal to 1, such as $\frac{1}{1} = \frac{2}{2}$). Multiplying both terms by the same number splits every part into equal smaller pieces, so the shaded area stays the same. For example, $\frac{2}{3} = \frac{(2 \times 2)}{(3 \times 2)} = \frac{4}{6}$.

WATCH FOR • Some students treat equivalence as something you make by adding. Equivalent fractions come from splitting or grouping parts, which means multiplying or dividing both terms by the same nonzero number.

TRANSFER • $\frac{3}{5} \neq \frac{6}{8}$, because $\frac{3}{5} = \frac{24}{40}$ and $\frac{6}{8} = \frac{30}{40}$. $\frac{3}{5} = \frac{6}{10}$, because both terms of $\frac{3}{5}$ are multiplied by 2.